Eur. Phys. J. D **22**, 109–116 (2003)
DOI: 10.1140/epjd/e2002-00225-9

THE EUROPEAN PHYSICAL JOURNAL D

Radiation reflection by a plasma with electron temperature anisotropy

G. Ferrante[1], M. Zarcone[1,a], and S.A. Uryupin[2]

[1] Istituto Nazionale per la Fisica della Materia, Dipartimento di Fisica e Tecnologie Relative, Viale delle Scienze, 90128 Palermo, Italy
[2] P.N. Lebedev Physical Institute, Leninsky pr. 53, 117924, Moscow, Russia

Received 21 May 2002 / Received in final form 21 August 2002
Published online 6 November 2002 –

Abstract. The reflection of a test electromagnetic wave normally impinging on a plasma surface is investigated within the formalism of the surface impedance. The plasma is assumed to possess an anisotropic two-temperature bi-Maxwellian electron velocity distribution function. The linearly polarized impinging wave during reflection transforms into an elliptically polarized one, the degree of ellipticity depending on the electron temperature anisotropy. Polarization modifications of the reflected wave are particularly important in the conditions of the anomalous skin-effect, when the influence of the wave magnetic field on the electron kinetics in the skin layer is strong. Relations are reported connecting the reflected wave basic parameters to those of the reflecting plasma surface, making possible, through the experimental determination of the reflected wave characteristics, to find the plasma electron concentration and the two effective temperatures.

PACS. 52.38.Dx Laser light absorption in plasmas (collisional, parametric, etc.) – 52.50.Jm Plasma production and heating by laser beams (laser-foil, laser-cluster, etc.)

1 Introduction

The properties of plasmas possessing an anisotropic electron distribution over velocities since several years are the subject of a considerable number of investigations (see, for instance, [1,2]). The interest to such an issue stems from the experimental evidence that in the interaction processes of intense ultra-short laser pulses with matter are easily created plasma media in which the electron velocity distribution function (EDF) is significantly anisotropic. In particular, anisotropic EDFs are formed in tunnel atom ionization [3–7]; in intense field inverse bremsstrahlung [8,9]; in plasmas exhibiting along some given direction a sufficient high degree of spatial non-uniformity [10]. Still another well investigated interaction process yielding anisotropic EDFs is that in which quasistatic electric and magnetic fields act upon the plasma electrons. Anisotropic EDFs are responsible for the appearance of new features in an entire series of plasma phenomena, as well as the cause of new ones. Among the latter, perhaps, one of the most interesting is the Weibel instability (see, for instance, [11–13]). Among anisotropic plasmas, a particular interest is attached to plasma with an anisotropic bi-Maxwellian EDF. Such a plasma exhibits unusual properties of inverse bremsstrahlung absorption [14]. In such a plasma the properties of harmonic generation of a pump field due to electron-ion collisions are found unusual also [15]. The same is true for plasma X-ray spectra [16].

In reference [17] we have reported on a theoretical treatment to deal with collisionless radiation absorption in the skin-layer of a plasma with an anisotropic bi-Maxwellian EDF, like that defined below in the beginning of Section 2. There it has been shown that the formed new anomalies of the collisionless absorption are due to the influence of the magnetic field on the electron kinetics in the skin-layer. In the present paper, we address another plasma phenomenon, caused by the EDF anisotropy. Namely, we investigate how the reflected wave polarization changes with respect to that of the incident wave when the reflection takes place at the surface of a plasma possessing an anisotropic bi-Maxwellian EDF. In the framework of the simplest, but frequently used model (see, also [18–22]), in which the electrons are assumed to undergo specular reflection at the plane plasma surface, in Section 2 we report the basic relations concerning: (i) the components of the surface impedance of an anisotropic plasma; (ii) the reflection coefficient; and (iii) the phase-shift, measuring the degree of the transformation of a linearly polarized incident wave into an elliptically polarized reflected wave. In Section 3 we report on a detailed analytical and numerical description of the impedance components imaginary parts as function of Δ, a quantity measuring the degree of

[a] e-mail: zarcone@unipa.it

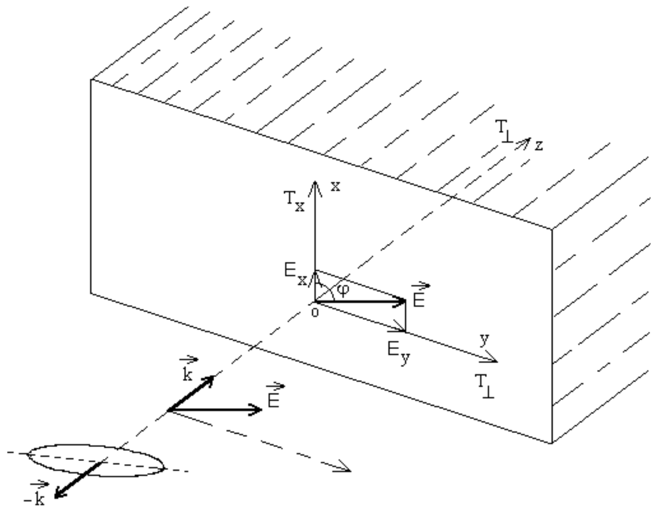

Fig. 1. A picture of reflection of a linearly polarized wave by a plasma with electron temperature anisotropy, and of the chosen geometry.

electron temperature anisotropy; and of δ, measuring the skin-effect anomaly degree in an isotropic plasma ($\varDelta$ and δ are defined below in Sects. 2 and 3). In Section 4 the behavior of the phase-shift $\varPsi$ *versus* $\varDelta$ and δ is investigated.

2 Basic relations

Let us consider the interaction of a linearly polarized electromagnetic wave of the form

$$\mathbf{E}_i(z,t) = \mathbf{E}\cos(\omega t - kz), \quad z < 0, \tag{1}$$

with plasma filling the half-space $z > 0$. A graphical representation of the interaction geometry is shown in Figure 1. In (1) $\mathbf{E} = (E_x, E_y, 0)$, and the frequency ω is related to the wave number k by the relation $\omega = kc$, where c is the speed of light. The frequency ω of the incident wave is assumed much smaller than the electron plasma frequency $\omega_L = \sqrt{4\pi e^2 N/m}$, where e, m and N are, respectively, the electron charge, mass and density. Further, it is assumed that the plasma EDF is given by an anisotropic bi-Maxwellian function with the symmetry axis along the Ox-direction, *i.e.*

$$F = \left(\frac{m}{2\pi}\right)^{3/2} \frac{N}{T_\perp \sqrt{T_x}} \exp\left[-\frac{mv_x^2}{2T_x} - \frac{m}{2T_\perp}\left(v_y^2 + v_z^2\right)\right]. \tag{2}$$

The choice of this EDF implies that the electron motion along the Ox-axis is characterized by the effective temperature (average kinetic energy) T_x, while by the temperature $T_\perp$ that in the yOz-plane. The temperatures are given in energy units. As known, a plasma with anisotropic EDF may be unstable against the development of the Weibel instability, yielding magnetic field generation and inverse influence of the generated magnetic field on the electron velocity distribution. Different regimes of Weibel instability occurring at different values of temperature anisotropy were studied in references [11–13]. Here we consider time intervals shorter than the inverse growth rate of the Weibel instability and accordingly we exclude from our consideration the EDF distortion due to this instability. The influence of the binary electron collisions is neglected. It is, in particular, justified by the conditions defining the anomalous skin-effect, taking place in sufficiently hot plasmas (see, for instance, [17–22]). Similarly to the analysis given in [17–22], we confine our investigation to the simplest case, when electrons are specularly reflected by the plasma plane surface. Assuming that the plasma has a sharp plane boundary we exclude from our consideration the hydrodynamic ion motion which leads finally to a density gradient. Such approximation is reasonable for time intervals smaller than the ratio of the effective skin depth to the acoustic velocity [18]. As the incident wave has two components E_x and E_y, the characteristics of its interaction with the anisotropic plasma are described by the two components Z_x and Z_y of the surface impedance. Following the usual procedure to describe the reflected wave and the field inside the plasma, for the impedance components we find (see Appendix)

$$Z_\alpha = -\frac{2\mathrm{i}}{\pi} k \int_0^\infty \frac{\mathrm{d}q}{q^2 - k^2 \epsilon_\alpha(\omega/qv_T)}, \quad (\alpha = x, y), \tag{3}$$

where $v_T = \sqrt{T_\perp/m}$. The component $\epsilon_x(\omega/qv_T)$ of the dielectric function entering the expression (3) is given by

$$\begin{aligned}\epsilon_x(\xi) = 1 - \varDelta\frac{\omega_L^2}{\omega^2} - \frac{\omega_L^2}{\omega^2}(1-\varDelta) \\ \times \left\{\xi \exp\left(-\frac{\xi^2}{2}\right)\int_0^\xi \mathrm{d}t \exp\left(\frac{t^2}{2}\right)\right. \\ \left. - \mathrm{i}\sqrt{\frac{\pi}{2}}\xi \exp\left(-\frac{\xi^2}{2}\right)\mathrm{sign}\ q\right\}, \quad \varDelta = 1 - T_x/T_\perp. \end{aligned} \tag{4}$$

The component $\epsilon_y(\omega/qv_T)$ too is described by an expression like (4), where, however, one must put $\varDelta = 0$. The surface impedance components (3) define the absorption and reflection coefficients A and R of a plasma with anisotropic bi-Maxwellian EDF according to the relations

$$A = 1 - R, \quad R = |R_x|^2 \cos^2\varphi + |R_y|^2 \sin^2\varphi, \tag{5}$$

$$R_\alpha = \frac{(Z_\alpha - 1)}{(Z_\alpha + 1)} = |R_\alpha| \exp(\mathrm{i}\varPsi_\alpha), \tag{6}$$

where φ is the angle between the incident wave polarization vector $\mathbf{E}$ and the Ox-axis, $\varPsi_\alpha$ is the wave phase-shift at the reflection by the plasma surface. The components of the reflected wave are $E_{rx}(z,t)$ and $E_{ry}(z,t)$, which may be written as

$$E_{r\alpha}(z,t) = |R_\alpha| E_\alpha \cos(\varPsi_\alpha - \omega t - kz), \tag{7}$$

where $E_x = E\cos\varphi$, $E_y = E\sin\varphi$. Finally, the functions $|R_\alpha|$ and $\varPsi_\alpha$, are expressed through the real and imaginary parts Z'_α and Z''_α of the surface impedance components $Z_\alpha = Z'_\alpha + \mathrm{i}Z''_\alpha$ according to the relations

$$|R_\alpha| = \left[(Z'_\alpha - 1)^2 + (Z''_\alpha)^2\right]^{1/2} \left[(Z'_\alpha + 1)^2 + (Z''_\alpha)^2\right]^{-1/2}, \tag{8}$$

$$\Psi_\alpha = \arctan\left[\frac{Z''_\alpha}{Z'_\alpha - 1}\right] - \arctan\left[\frac{Z''_\alpha}{Z'_\alpha + 1}\right]. \tag{9}$$

According to relations (7–9) the link between the reflected wave and the incident one is fully determined by the real and imaginary parts of the impedance. As a consequence of the Landau damping, the intensity of the reflected wave is smaller than the intensity of the incident one. The intensity decrease of the reflected wave is measured by the decrease of the function $|R_\alpha|$ with respect to unity. Another important effect is related to the circumstance that different components of the incident wave are reflected by the anisotropic plasma with different phase-shifts, $\Psi_x \neq \Psi_y$. Due to the difference between Ψ_x and Ψ_y, the reflected wave polarization differs as compared to that of the incident wave. Namely, the linearly polarized incident wave (1) is reflected by the anisotropic plasma as an elliptically polarized wave (7) (see Fig. 1). Under these conditions, the ellipticity degree of the reflected wave is characterized by the phase-shift difference

$$\Psi = \Psi_x - \Psi_y. \tag{10}$$

In the physical conditions under consideration here, the real and imaginary parts of the impedance components in absolute value show small departures from unity (see Ref. [17] and below the asymptotic formulae (16–24) and Figs. 2 and 3). It allows to simplify the relations (8, 9), and to write the following approximate expressions for the reflection coefficient R (5), and phase-shift Ψ (10)

$$R = 1 - A \simeq 1 - 4Z'_x \cos^2\varphi - 4Z'_y \sin^2\varphi, \tag{11}$$

$$\Psi \simeq 2Z''_y - 2Z''_x. \tag{12}$$

According to (11) the reflection (absorption) coefficient is basically determined by the real parts of the impedance components. The functions Z'_α and A (as given by (11)) has been analyzed by us previously [17]. At the contrary, the phase-shift is determined by the difference of the imaginary parts. The following two sections are devoted to the investigation of the functions Z''_α and Ψ (12).

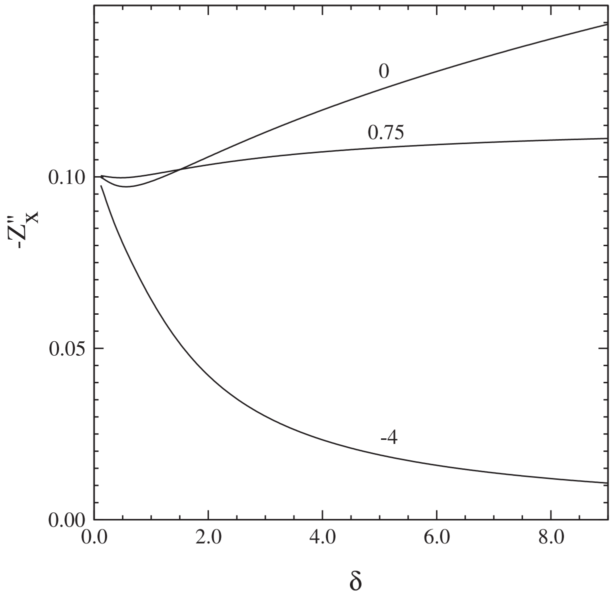

Fig. 2. The imaginary part of the impedance component Z''_x *versus* the parameter $\delta = v_T\omega_L/\omega c$. The different curves correspond to three values of the parameter $\varDelta = 1 - T_x/T_\perp$ characterizing the degree of temperature anisotropy $\varDelta$: 0 ($T_\perp = T_x$); 0.75 ($T_\perp = 4T_x$); -4 ($T_x = 5T_\perp$).

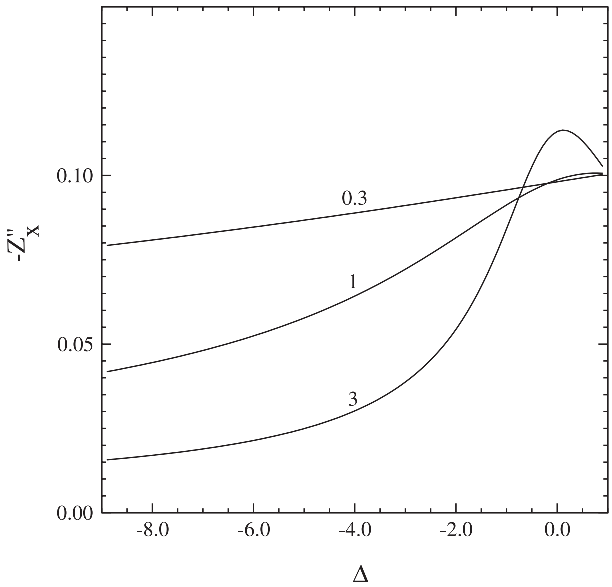

Fig. 3. The same function as in Figure 2, but *versus* $\varDelta = 1 - T_x/T_\perp$ for three values of the parameter δ: 0.3, 1, 3. $\Omega = \omega/\omega_L = 0.1$.

3 Impedance imaginary parts

Let us consider now the behavior of the imaginary parts of the impedance components in different limiting cases. With this aim we introduce the notations

$$\Omega = \omega/\omega_L \ll 1, \quad \delta = v_T\omega_L/\omega c, \tag{13}$$

where δ is the parameter characterizing the anomaly degree of the skin-effect. Then, from (3) and (4) for the component Z''_x we have

$$Z''_x = -\frac{2}{\pi}\delta\Omega \times \int_0^\infty \mathrm{d}x \frac{\mathrm{Re}(x)}{\left[\mathrm{Re}^2(x) + \frac{\pi}{2}(1-\varDelta)^2\delta^4 x^6 \exp(-x^2)\right]}, \tag{14}$$

with

$$\mathrm{Re}(x) = 1 - x^2\delta^2 \left[\Omega^2 - \Delta - (1-\Delta)\, x \exp\left(-\frac{x^2}{2}\right) \int_0^x \mathrm{d}t \exp\left(\frac{t^2}{2}\right) \right]. \tag{15}$$

The corresponding expression for Z_y'' follows from (14) and (15) if we put there $\Delta = 0$. As we are interested in the conditions when $\Omega = \omega/\omega_L \ll 1$ and $\Omega\delta = v_T/c \ll 1$, in the asymptotic expressions to be obtained below the dependence of $\mathrm{Re}(x)$ from Ω^2 will be neglected.

Below we analyze the behavior of the impedance imaginary part as a function of the parameters Δ and δ. As stated, the parameter $\Delta = 1 - T_x/T_\perp$ is a measure of the electron temperature degree of anisotropy. To highly anisotropic plasmas with EDF elongated along the Ox-axis correspond $T_x \gg T_\perp$, $\Delta < 0$ and $|\Delta| \gg 1$. To highly anisotropic plasmas, in which the electrons are much hotter in the yOz-plane correspond instead $T_\perp \gg T_x$, and $\Delta \sim 1$. To weakly anisotropic plasmas correspond instead $|T_x - T_\perp| \ll T_\perp$, and $|\Delta| \ll 1$.

As far as the parameter $\delta = \omega_L v_T/\omega c$ is concerned, we point out that it characterizes the degree of anomaly of the skin-effect in an *isotropic* plasma, when $T_x = T_\perp$. For such a plasma, the condition $\delta < 1$ corresponds to the high-frequency skin-effect, when the distance gone through by thermal electrons in a field period v_T/ω is smaller than the skin layer depth c/ω_L. The condition $\delta > 1$, instead, corresponds to $v_T/\omega > c/\omega_L$, when the anomalous skin-effect takes place. In an anisotropic plasma the relation between $v_T = \sqrt{T_\perp/m}$ and c/ω_L, and with it the parameter δ itself, rigorously speaking, can not serve alone to define the transition condition from the high-frequency to the anomalous skin-effect. As a matter of fact, in an anisotropic plasma, because of the influence of the magnetic field on the electron motion in the skin layer, the electron thermal motion along the Ox-axis becomes very important (see Appendix). In this context, the transition condition from high-frequency to anomalous skin-effect may be also connected to the relation between $v_{T_x} = \sqrt{T_x/m}$ and the skin layer depth, which, by the way, may not coincide with c/ω_L. It is not difficult to carry out the analysis of the anomaly degree of the skin-effect for an anisotropic plasma. However, as it is not necessary to evidence the peculiar features of the incident wave reflection and absorption, which are attainable experimentally, such an analysis will be not pursued here.

3.1 Limiting forms of Z_y'' for the cases when $\delta \ll 1$ and $\delta \gg 1$

We first consider the behavior of the simpler function Z_y''. Assuming $\delta \ll 1$, in (14) the largest contributions to the integral over x comes from values $x \simeq 1/\delta \gg 1$. Then, using the approximate relation $\mathrm{Re}(x) \simeq 1 + x^2\delta^2$ and neglecting the second exponentially small term in the denominator of (14) from the latter we find

$$Z_y'' \simeq -\Omega, \quad \delta \ll 1. \tag{16}$$

If, instead, $\delta \gg 1$, in (18) the integral over x diverges for $x \lesssim \delta^{-2/3} \ll 1$. For such small x, it is possible to approximate $\mathrm{Re}(x)$ to unity. Then we have

$$\begin{aligned} Z_y'' &\simeq -\frac{2}{\pi}\delta\Omega \int_0^\infty \frac{\mathrm{d}x}{1 + \delta^4 x^6 \frac{\pi}{2}} \\ &= -\frac{2}{3}\left(\frac{2}{\pi}\right)^{1/6} \Omega\delta^{1/3}, \quad \delta \gg 1. \end{aligned} \tag{17}$$

3.2 Limiting forms of Z_x'' for the cases when $\delta \ll 1$ and $\delta \gg 1$

Let us consider now Z_x''. We start the analysis with the case when the anomaly parameter is very small ($\delta \ll 1$). If the parameter Δ, characterizing the degree of temperature anisotropy, satisfies the condition $\delta\sqrt{1-\Delta} \ll 1$, then in (14) the largest contribution to the integral over x comes from $x \simeq 1/\delta \ll 1$, and the function Z_x'' approximately is the same as Z_y'' (16),

$$Z_x'' \simeq -\Omega, \quad \delta \ll 1, \quad \delta\sqrt{1-\Delta} \ll 1. \tag{18}$$

If, additionally, $\Delta < 0$ and $\delta\sqrt{|\Delta|} \gg 1$, then the primary contribution to Z_x'' (14) comes from $x \approx 1/\sqrt{|\Delta|}$,

$$Z_x'' \simeq -\frac{2}{\pi}\delta\Omega \int_0^\infty \frac{\mathrm{d}x}{1 + |\Delta|\delta^2 + x^2\delta^2} \simeq -\frac{\Omega}{\delta\sqrt{|\Delta|}}, \quad \Delta < 0, \quad \delta \ll 1, \quad \delta\sqrt{|\Delta|} \gg 1. \tag{19}$$

In the limit of high values of the anomaly parameter ($\delta \gg 1$), the number of interesting limiting cases becomes larger. The first of them corresponds to the situation, when $1 - \Delta \ll 1$; *i.e.* to the situation when one has a highly anisotropic plasma with $T_x \ll T_\perp$. For such δ and Δ in (14) are significant only the values of $x \simeq 1/\delta \ll 1$, and for Z_x'' we obtain a result similar to (18):

$$Z_x'' \simeq -\Omega, \quad \delta \gg 1, \quad 1 - \Delta \ll 1. \tag{20}$$

As the parameter Δ decreases one has an increase of the x values giving important contributions to the integral (14), according to the relation $x \sim 1/\delta\sqrt{\Delta}$ if Δ satisfies the inequalities $1 \gg \Delta \gg \delta^{-2/3}$. In such conditions from (14) we find

$$Z_x'' \simeq -\frac{\Omega}{\sqrt{\Delta}}, \quad 1 \gg \Delta \gg \delta^{-2/3}. \tag{21}$$

In the case of relatively weak temperature anisotropy, when $|\Delta| \ll \delta^{-2/3} \ll 1$, the largest contribution to Z_x'' (14) comes from $x \simeq \delta^{-2/3} \ll 1$, and the function Z_x'' has the same asymptotic expression (17) as Z_y'':

$$Z_x'' \simeq -\frac{2}{3}\left(\frac{2}{\pi}\right)^{1/6} \Omega\delta^{1/3}, \quad 1 \gg \delta^{-2/3} \gg \Delta. \tag{22}$$

In the domain where $\Delta < 0$ and in absolute value is relatively small, $1 \gg |\Delta| \gg \delta^{-2/3}$, for calculation of Z''_x (14), we may use the approximate expression $\mathrm{Re}(x) \simeq 1 - |\Delta| x^2 \delta^2$. Then, from (14) we find

$$Z''_x \simeq -\frac{2}{\pi}\delta\Omega \int_0^\infty \frac{1-|\Delta|x^2\delta^2}{(1-|\Delta|x^2\delta^2)^2 + \frac{\pi}{2}\delta^4 x^6 \exp(-x^2)} \mathrm{d}x$$
$$\simeq -\sqrt{\frac{\pi}{2}} \frac{\Omega}{\delta\Delta^2}, \quad \Delta < 0, \quad 1 \gg |\Delta| \gg \delta^{-2/3}. \qquad (23)$$

Finally, for $\Delta < 0$ and large value of $|\Delta|$ from (14) we arrive to a result of the form (19)

$$Z''_x \simeq -\frac{\Omega}{\delta\sqrt{\Delta}}, \quad \Delta < 0, \quad |\Delta| \gg 1, \quad \delta \gg 1. \qquad (24)$$

Comparing (19) and (24) we can see that the common conditions of their validity are the inequalities $|\Delta| \gg \max(1, \delta^{-2})$, $(\Delta < 0)$.

3.3 Results of the numerical calculations

The results of the numerical calculations of the impedance imaginary part Z''_x are reported in Figures 2 and 3. Figure 2 shows plots of the function $-Z''_x$ *vs.* δ, parameter measuring the degree of anomaly of the skin-effect. The three curves in Figure 2 corresponds to three values of the temperature anisotropy: $T_x = T_\perp$ ($\Delta = 0$), $T_\perp = 4T_x$ ($\Delta = 0.75$), $T_x = 5T_\perp$ ($\Delta = -4$). The curve corresponding to $\Delta = 0$ describes the function $-Z''_y$ as well. The behavior of this latter curve is in agreement with that following from the asymptotic expressions (16, 17). The curve with $\Delta = 0.75$ shows a small modification in the function $-Z''_x$, in agreement with formulae (18, 20). The curve corresponding to $\Delta = -4$, for small δ values, behaves as $-Z''_x \simeq \Omega = 0.1$, in agreement with (18), while, with the increase of δ, it shows a decreasing behavior according to formulae (19) and (24).

Figure 3 shows plots of $-Z''_x$ *vs.* $\Delta = 1 - T_x/T_\perp$ for three values of $\delta = 0.3$, 1, 3. The curve with $\delta = 0.3$ shows a small modification of $-Z''_x$, which does not contradict formulae (18, 19). These latter formulae (or formulae (20, 24)) allow to qualitatively understand the behavior of the $-Z''_x$ curve with $\delta = 1$. The non-monotonic behavior of the $-Z''_x$ curve with $\delta = 3$ corresponds to formulae (16, 21–24). In fact, according to (16) for Δ values close to unity, one has $-Z''_x \simeq \Omega = 0.1$, the function $-Z''_x$ increases according to (21) and reaches its maximum in the region $|\Delta| \ll \delta^{-2/3}$, which corresponds to formula (22). Further, in agreement with formula (23), for $1 > |\Delta| > \delta^{-2/3}$, the function $-Z''_x$ shows a relatively rapid decrease proportional to Δ^{-2}. Finally, as $|\Delta| > 1$, a domain is entered where formula (24) holds, which predict a slow decrease of the impedance imaginary part with the increase of the electron temperature anisotropy.

4 Phase-shift at reflection

After having determined the impedance imaginary parts, we move to discuss how the phase-shift of the reflected wave depends on the plasma parameters. First of all, using the asymptotic formulae (16–24) derived in the previous Section 3, we give a summary of approximate formulae of the phase-shift Ψ (12) for different domains of the plasma parameters:

$$\Psi \simeq 0, \qquad |\Delta| \ll \max(\delta^{-2}, \delta^{-2/3}), \qquad (25)$$

$$\Psi \simeq -2\Omega + \frac{2\Omega}{\delta\sqrt{|\Delta|}}, \quad \delta \ll 1, \quad \delta\sqrt{|\Delta|} \gg 1, \qquad (26)$$

$$\Psi \simeq 2\Omega - \Psi_m, \qquad \delta \gg 1, \quad 1 - \Delta \ll 1, \qquad (27)$$

$$\Psi \simeq \frac{2\Omega}{\sqrt{|\Delta|}} - \Psi_m, \qquad 1 \gg \Delta \gg \delta^{-2/3}, \qquad (28)$$

$$\Psi \simeq \frac{\pi}{2}\frac{\Omega}{\delta\Delta^2} - \Psi_m, \qquad 1 \gg |\Delta| \gg \delta^{-2/3}, \quad \Delta < 0, \qquad (29)$$

$$\Psi \simeq \frac{2\Omega}{\delta\sqrt{|\Delta|}} - \Psi_m, \qquad |\Delta| \gg 1, \quad \delta \gg 1, \qquad (30)$$

where Ψ_m stands for the maximum absolute value of the phase-shift

$$\Psi = \frac{4}{3}\left(\frac{2}{\pi}\right)^{1/6} \Omega\delta^{1/3} = \frac{4}{3}\left(\frac{2}{\pi}\right)^{1/6} \left(\frac{v_T \omega^2}{\omega_L^2 c}\right)^{1/3} \cdot \qquad (31)$$

According to (25) the phase-shift is close to zero when the electron temperature anisotropy is relatively small. As the degree of anisotropy is increased, the absolute value of the phase-shift too increases. For $\delta \ll 1$ the phase-shift reaches the value -2Ω (26) for rather large temperature anisotropy, when $|\Delta| \gg \delta^{-2} \gg 1$ or $T_x \gg T_\perp\delta^{-2} \gg T_\perp$. When, instead, $\delta \gg 1$, one has that: (i) the maximum phase-shift absolute value Ψ_m becomes significantly larger than 2Ω; and (ii) Ψ_m is reached for relatively small temperature anisotropy, $|T_x - T_\perp| \gg T_\perp\delta^{-2/3}$. That the phase-shift is larger in the case of large δ values is traced back to the fact that than larger the skin-effect degree of anomaly, than stronger the wave magnetic field influence on the electron kinetics in the skin-layer. Figure 4 reports plots of the phase-shift Ψ *versus* δ for three values of Δ: 0.75 ($T_\perp = 4T_x$), -1 ($T_x = 2T_\perp$), -4 ($T_x = 5T_\perp$). For small δ all the curves are close to zero, in agreement with formula (25). Far from $\delta = 0$, the behavior of the function Ψ depends on the value Δ. When $T_\perp = 4T_x$, ($\Delta = 0.75$), due to the Z''_x weak variation for $\delta \le 9$ (see Fig. 2), the Ψ dependence on δ is essentially controlled by the function $2Z''_y$ shifted by 2Ω along the ordinate axis. If $T_x = 5T_\perp$ ($\Delta = -4$) the behavior of the curve in Figure 4 corresponds to formulae (25, 26, 30). Finally, the curve of Figure 4 with $\Delta = -1$ (or $T_x = 2T_\perp$) is qualitatively described by formulae (25, 29, 30). As a whole, Figure 4 shows that than larger δ and the ratio $T_x/T_\perp$, than larger the absolute value of the function Ψ too. Figure 5 reports plots showing the phase-shift Ψ *vs.* Δ for three values of δ: 0.3, 1 and 3. The behavior of the curves shown in Figure 5 bears resemblance with that of the function $-2Z''_x$ (see Fig. 3), with the only difference that all the curves are shifted along the ordinate axis by the amount $2Z''_y$, depending on the δ values. The function Ψ, analyzed numerically in this section, together with $|R_\alpha| \simeq 1 - 2Z'_\alpha$ (8),

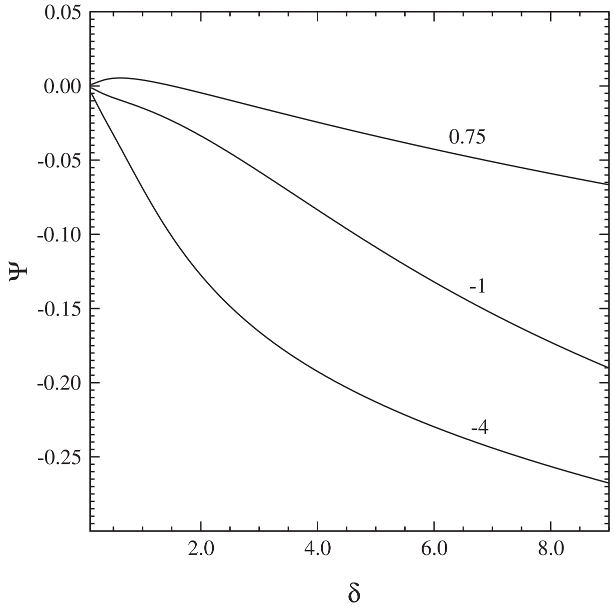

Fig. 4. The phase-shift of the reflected wave Ψ *versus* $\delta = v_T\omega_L/\omega c$ for three values of the electron temperature anisotropy: $\Delta = 0.75$ ($T_\perp = 4T_x$); $\Delta = -1$ ($T_x = 2T_\perp$); $\Delta = -4$ ($T_x = 5T_\perp$).

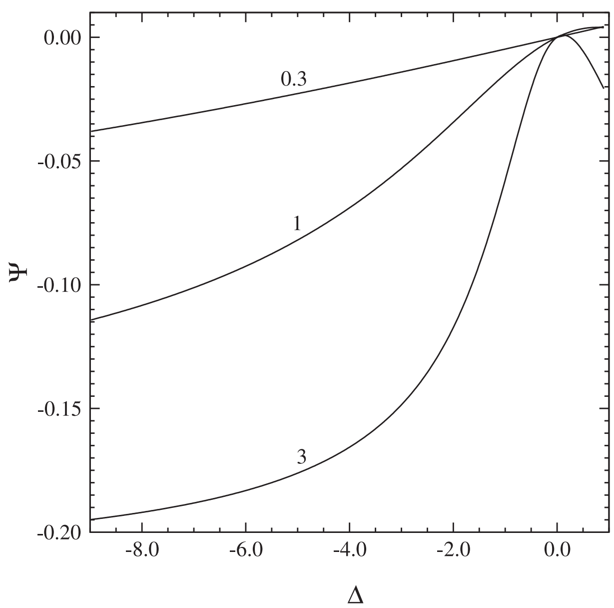

Fig. 5. The same function as in Figure 4, *versus* electron temperature anisotropy $\Delta = 1 - T_x/T_\perp$. The different curves correspond to three values of the parameter $\delta = v_T\omega_L/\omega c$: 0.3, 1, 3.

fully determines the intensity and polarization of the reflected wave. Sometimes, to describe the elliptically polarized wave, the Stokes parameters are used (see, for instance, [24]). When the absolute values of the impedance components Z'_α and Z''_α are small, for the Stokes parameters approximately one has

$$S_0 = |R_x|^2 \cos^2\varphi + |R_y|^2 \sin^2\varphi \simeq 1 - A, \tag{32}$$

$$S_1 = |R_x|^2 \cos^2\varphi - |R_y|^2 \sin^2\varphi \\ \simeq \cos 2\varphi - 4Z'_x \cos^2\varphi + 4Z'_y \sin^2\varphi, \tag{33}$$

$$S_2 = |R_x||R_y| \sin 2\varphi \cos\Psi \\ \simeq (1 - 2Z'_x - 2Z'_y)\sin 2\varphi, \tag{34}$$

$$S_3 = |R_x||R_y| \sin 2\varphi \sin\Psi \simeq 2(Z''_y - Z''_x)\sin 2\varphi. \tag{35}$$

The relations (32–35), together with the real component of the surface impedance Z'_α, investigated in [17], and with the phase difference Ψ, considered above, provide full and exhaustive information on the reflected wave by a collisionless overdense plasma with an anisotropic bi-Maxwellian electron velocity distribution.

5 Conclusions

In this report we have shown how a linearly polarized wave, reflected by a plasma surface in which the electrons exhibit an anisotropic bi-Maxwellian velocity distribution, transforms into an elliptically polarized wave. The degree of ellipticity is given by the difference of the phase-shifts of two components of the reflected wave.

We have also shown, that, when the distance traversed by thermal electrons in a field period is less than the skin-depth $v_T/\omega \ll c/\omega_L$, the absolute value of the phase-shift difference is largest in a plasma with $v_{Tx}/\omega \gg c/\omega_L$, where $v_{Tx} = \sqrt{T_x/m}$ is the thermal velocity along the EDF anisotropy axis. This case corresponds to highly anisotropic plasmas with $T_\perp \ll T_x$. In the opposite case, when $v_T/\omega \gg c/\omega_L$, one has a stronger modification of the reflected wave polarization. In such a case the largest value of the phase-shift difference is found to exceed the corresponding value of the previous case by the factor $\delta^{1/3}$, which is much larger than 1. Besides, for its occurrence is only required a relatively small temperature anisotropy, $|T_x - T_\perp| \gg T_\perp \delta^{-2/3}$.

In the framework of the reported theoretical treatment it has been possible to connect the basic parameters of the reflected wave such as $|R_x|$, $|R_y|$, and Ψ to the parameters of the reflecting anisotropic bi-Maxwellian plasma $\Omega = \omega/\omega_L$, $\delta = \omega_L v_T/\omega c$ and $\Delta = 1 - T_x/T_\perp$. It means that in principle is possible to determine δ, Ω and Δ through the experimentally measurable quantities $|R_x|$, $|R_y|$, and Ψ. In its turn, it also means that one can determine the electron concentration and the two effective temperatures T_x and $T_\perp$. Experiments aimed at measuring the above quantities would be two-fold useful. On one hand, they would give information about plasmas with anisotropic EDF, which are still not enough studied; on the other hand, they could yield a better understanding of different aspects of the processes of reflection and absorption of a test wave by an highly non-equilibrium plasma, and show the ways to improve their theoretical description.

This work is part of the research activity of the Italian-Russian Forum of Laser Physics and Related Technologies and was supported by the Russian Fund for Basic Research (project N02-02-16047) and the Russian-Italian Agreement for Scientific Collaboration. The authors wish to acknowledge also the support by the Palermo University through the International Relations Fund.

Appendix

Here we give the derivation of the surface impedance components and show how the wave magnetic field affects the electron motion. Let us consider the interaction of the E_x component with the plasma.The field inside the plasma has the form

$$\frac{1}{2}E_x(z)\exp(-\mathrm{i}\omega t) + \text{c.c.}, \qquad z > 0. \tag{A.1}$$

To define the field $E_x(z)$, from the Maxwell equations we have

$$\frac{\mathrm{d}^2}{\mathrm{d}z^2}E_x(z) + \frac{\omega^2}{c^2}E_x(z) = -\frac{4\pi\mathrm{i}\omega}{c^2}\int \mathrm{d}\mathbf{v} v_z \delta f. \tag{A.2}$$

The perturbation δf to the electron distribution function F is found from the kinetic equation

$$-\mathrm{i}\omega\delta f + v_z\frac{\mathrm{d}}{\mathrm{d}z}\delta f = \frac{e}{m}\Bigg\{E_x(z)\frac{\partial F}{\partial v_x} + \frac{1}{c}B_y(z)\left[v_x\frac{\partial F}{\partial v_z} - v_z\frac{\partial F}{\partial v_x}\right]\Bigg\} = -S_x(z, v_z), \tag{A.3}$$

where $B_y(z)$ is the magnetic field component in the plasma created by the impinging wave

$$B_y(z) = -\mathrm{i}\frac{c}{\omega}\frac{\mathrm{d}}{\mathrm{d}z}E_x(z). \tag{A.4}$$

In plasma with isotropic EDFs, the term in (A.3) containing $B_y(z)$ goes to zero. In other words, in isotropic plasmas, the magnetic field does not affect the electrons kinetics in the skin layer. At the contrary, in plasmas with anisotropic EDFs like the bi-Maxwellian F, the term containing $B_y(z)$ is not zero. Physically, it implies that the anisotropic electron distribution over velocities creates the conditions for the magnetic field $B_y(z)$ to rotate the electrons from one degree of freedom to the other. Because of this the magnetic field contributes to determine δf, *i.e.* to influence significantly the electron kinetics. Besides, in the skin-effect conditions, the magnetic field in absolute value considerably exceeds the electric field, according to the inequality $c/\omega \gg |\mathrm{d}\ln E_x(z)/\mathrm{d}z|^{-1}$. The joint manifestation of both these causes is responsible for the appearance of new optical properties in plasmas with anisotropy in the electron distribution function. Further, we consider the simplest boundary conditions on the plasma surface. Namely, we assume that electrons are specularly reflected by the plasma boundary. Provided these conditions are fulfilled, equation (A.3) gives

$$\delta f = \frac{1}{v_z}\int_z^\infty \mathrm{d}z' S_x(z', v_z)\exp\left[\mathrm{i}\frac{\omega}{v_z}(z-z')\right], \qquad v_z < 0; \tag{A.5}$$

$$\delta f = -\frac{1}{v_z}\int_0^z \mathrm{d}z' S_x(z', v_z)\exp\left[\mathrm{i}\frac{\omega}{v_z}(z-z')\right] - \frac{1}{v_z}\int_0^\infty \mathrm{d}z' S_x(z', -v_z)\exp\left[\mathrm{i}\frac{\omega}{v_z}(z+z')\right], \quad v_z > 0; \tag{A.6}$$

Substituting the perturbation δf (A.5, A.6) into the r.h.s. of equation (A.2) and performing the Fourier transform over z, as it is usually done when specular reflection conditions are assumed (see, for instance, [23]), from (A.2) we obtain

$$E_x(z) = -\mathrm{i}\frac{k}{\pi}\frac{E_x(+0)}{Z_x}\int_{-\infty}^{\infty}\frac{\mathrm{d}q}{q^2 - k^2\varepsilon_x(\omega/qv_T)}\exp(\mathrm{i}qz), \quad z > 0; \tag{A.7}$$

where $E_x(+0)$ is the electric field on the plasma boundary. In writing the expression (A.7), we have used the notation Z_x to indicate the component of the surface impedance $Z_x = E_x(+0)/B_y(+0)$, giving the ratio between the electric and magnetic fields on the plasma surface. In (A.7) the $\varepsilon_x(\omega/qv_T)$ is given by formula (4). Finally, equations (A.4) and (A.7) gives us the formula (3) for Z_x. The expression for Z_y follows from Z_x, if we let $T_x = T_\perp$.

References

1. R.C. Davidson, in *Basic Plasma Physics*, edited by A.A. Galeev, R.N. Sudan (North-Holland, Amsterdam, 1983), Vol. 1, p. 519
2. G. Ferrante, M. Zarcone, S.A. Uryupin, Plasma Sources Sci. Technol. **10**, 318 (2001)
3. P.B. Corkum, N.H. Burnett, F. Brunel, Phys. Rev. Lett. **62**, 1259 (1989)
4. N.B. Delone, V.P. Krainov, J. Opt. Soc. Am. B **8**, 1207 (1991).
5. W.P. Leemans, C.E. Clayton, W.B. Mori, K.A. Marsh, A. Dyson, C. Joshi, Phys. Rev. Lett. **68**, 321 (1992)
6. J.C. Kieffer, J.P. Matte, H. Pepin, M. Chaker, Y. Beaudoin, T.W. Johnston, C.Y. Chien, S. Coe, G. Mourou, J. Dubau, Phys. Rev. Lett. **68**, 480 (1992)
7. V.P. Silin, JETP **90**, 805 (2000)
8. B.N. Chichkov, S.A. Shumsky, S.A. Uryupin, Phys. Rev. A **45**, 7475 (1992)
9. P. Porshnev, S. Bivona, G. Ferrante, Phys. Rev. E **50**, 3943 (1994)
10. J.C. Kieffer, J.P. Matte, M. Chaker, Y. Beaudoin, C.Y. Chien, S. Coe, G. Mourou, J. Dubau, M. Inal, Phys. Rev. E **48**, 4648 (1993)
11. V.P. Silin, S.A. Uryupin, JETP **84**, 59 (1997)

12. A. Bendib, K. Bendib, A. Sid, Phys. Rev. E **55**, 7522 (1997)
13. G. Ferrante, M. Zarcone, S.A. Uryupin, Phys. Rev. E **64**, 046408 (2001)
14. G. Ferrante, M. Zarcone, S.A. Uryupin, Phys. Plasmas **8**, 4745 (2001)
15. G. Ferrante, M. Zarcone, S.A. Uryupin, Laser Part. Beams **20**, (2002, in press)
16. H. Yoneda, N. Hasegawa, S. Kawana, K. Ueda, Phys. Rev. E **56**, 988 (1997)
17. G. Ferrante, M. Zarcone, S.A. Uryupin, Eur. Phys. J. D **19**, 349 (2002)
18. W. Rozmus, V.T. Tikhonchuk, Phys. Rev. A **42**, 7401 (1990)
19. P. Mulser, S. Pfalzner, F. Cornolti, in *Laser Interaction with Matter*, edited by G. Velarde, E. Minguez, J.M. Perlado (World Scientific, Singapore, 1989), p. 142
20. A.A. Andreev, E.G. Gamaly, V.N. Novikov, A.N. Semakhin, V.T. Tikhonchuk, JETP **74**, 963 (1992)
21. T.-Y. Brian Yang, W.L. Kruer, R.M. More, A.B. Langdon, Phys. Plasmas **2**, 3146 (1995)
22. T.-Y. Brian Yang, W.L. Kruer, A.B. Langdon, T.W. Johnston, Phys. Plasmas **3**, 2702 (1996)
23. E.M. Lifshitz, L.P. Pitaevskii, *Physical Kinetics* (Pergamon, Oxford, 1981), Chap. 9
24. S.A. Akhmanov, S.Yu. Nikitin, *Physical Optics* (Moscow State University, Moscow, 1998), p. 34 (*in russian*)